Then solving for $c_1'(t)$ and $c_2'(t)$ and integrating, we find finally:

$$y_p(t) = \int_0^t \frac{1}{\omega}\sin(\omega t - \omega s)\cdot f(s)\,ds =$$

$$= \begin{cases} \int_0^t \frac{1}{\omega}[\sin(\omega t-\omega s)]\cdot(-s)\,ds & \text{, if } 0<t<1 \\ \int_0^1 \frac{1}{\omega}[\sin(\omega t-\omega s)](-s)\,ds + \int_1^t \frac{1}{\omega}[\sin(\omega t-\omega s)](s-2)\,ds & \text{, if } 1\le t<2 \\ \int_0^1 \frac{1}{\omega}[\sin(\omega t-\omega s)](-s)\,ds + \int_1^2 \frac{1}{\omega}[\sin(\omega t-\omega s)](s-2)\,ds + 0 & \text{, if } t\ge 2 \end{cases}$$

$$y_p(t) = \begin{cases} \dfrac{\sin(\omega t) - \omega t}{\omega^3}, & \text{if } 0<t<1 \\ \dfrac{\sin(\omega t) - 2\sin(\omega t-\omega) + \omega(t-2)}{\omega^3}, & \text{if } 1\le t<2 \\ \dfrac{\sin(\omega t) - 2\sin(\omega t-\omega) + \sin(\omega t - 2\omega)}{\omega^3}, & \text{if } t\ge 2 \end{cases}$$

Note that $y_p(0) = 0 = y_p'(0)$ and therefore

$$y(t) = A\cos(\omega t) + \frac{B}{\omega}\sin(\omega t) + y_p(t).$$

② a) S is 2-dimensional.

b) It must be shown that there exists at least 2 independent elements in S (this is lemma 5) and, it must be shown that there are not more than 2 independent elements in S (this is lemma 4)

c) Easy to see that $W[y_1, y_2](0) = -1 \neq 0$, hence y_1, y_2 are indep. by lemmas 1 and 2.

d) Easy to see $W[y_1, y_2](0) = 0$, hence y_1, y_2 are dependent.

③ a) ~~Easy to~~ Characteristic eq: $r^2 + pr + 1 = 0 \Rightarrow r_{1,2} = \dfrac{-p \pm \sqrt{p^2-4}}{2}$

$p = 0 \Rightarrow r_{1,2} = \pm i \Rightarrow y(t) = c_1\cos t + c_2 \sin t$

But $y(0) = 0 = c_1$

$y'(0) = 1 = c_2$

$\Rightarrow y(t) = \sin t$

The solution oscillates without decay and

$$E(t) = 4y^2(t) + y'(t)^2 = 4\sin^2 t + \cos^2 t = 3\sin^2 t + \sin^2 t + \cos^2 t =$$

$$= 1 + 3\sin^2(t) \ge 1 \quad \text{oscilates.}$$