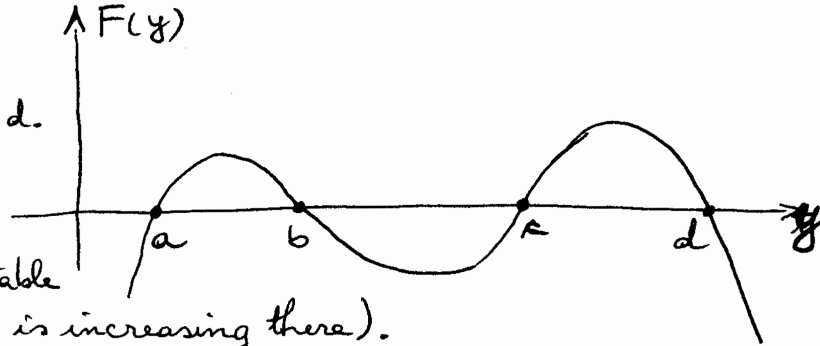
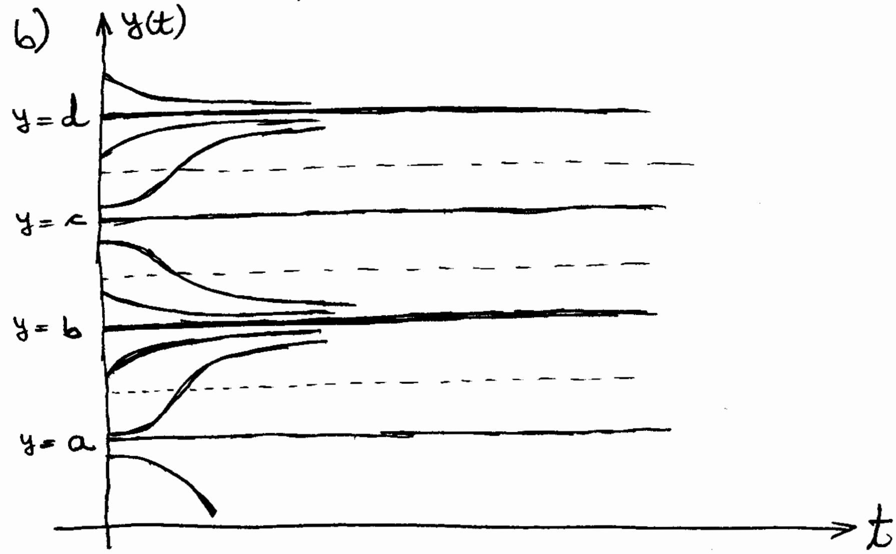
③



a) There are critical points at $y = a, b, c, d$.
The critical points $y = b, d$ are stable since $F'(y)$ is negative there.
The critical points $y = a, c$ are unstable since $F'(y)$ is positive there ($F(y)$ is increasing there).

b)



c) On any solution curve that originates at $y(0)$ between $y = b$ and $y = c$ we have $F(y(t)) < 0$ which means that $y'(t) < 0$ along any such curve. (since $y'(t) = F(y(t))$). Then $y(t)$ is decreasing along the curve which must therefore approach the value b. The curve can never cross the line $y = b$, however, since at any crossing point we would have $y = b$ and $y' < 0$ which contradicts the equation which asserts that $y' = 0$ when $y = b$.

④ a) The auxiliary eq is: $r^2 + 2pr + 36 = 0$. The solutions are $r_{1,2} = -p \pm \sqrt{p^2 - 36}$.
- when $0 \le p < 6 = P_0$, the roots are complex and the sol. to diff. eq. oscillates.
- when $p \ge 6 = P_0$, the roots are real and the sol. is decaying exponential (no longer oscillates)

b) When $p = \frac{1}{2} P_0 = \frac{1}{2} \cdot 6 = 3 \Rightarrow r_{1,2} = -3 \pm \sqrt{-27} = -3 \pm i\,\underbrace{3\sqrt{3}}_{\beta} \Rightarrow$

$\Rightarrow y(t) = c_1 e^{-3t} \cos(\beta t) + c_2 e^{-3t} \sin(\beta t)$ and $\begin{cases} y(0) = c_1 \\ y'(0) = -3c_1 + \beta c_2 \end{cases}$

Hence i) $c_1 = 1$, $c_2 = \frac{3}{\beta}$
ii) $c_1 = 0$, $c_2 = 1/\beta$

c) When $p = P_0 = 6 \Rightarrow r_1 = r_2 = -6 \Rightarrow y(t) = c_1 e^{-6t} + c_2 t e^{-6t}$ and $\begin{cases} y(0) = c_1 \\ y'(0) = -6c_1 + c_2 \end{cases}$

Hence i) $c_1 = 1$, $c_2 = 6$; ii) $c_1 = 0$, $c_2 = 1$.

d) When $p = \frac{3}{2} P_0 = 9 \Rightarrow r_{1,2} = -9 \pm \sqrt{45} \Rightarrow y(t) = c_1 e^{(-9-\sqrt{45})t} + c_2 e^{(-9+\sqrt{45})t}$

and $\begin{cases} y(0) = c_1 + c_2 \\ y'(0) = (-9-\sqrt{45})c_1 + (-9+\sqrt{45})c_2 \end{cases}$ Hence i) $\begin{cases} c_1 = \frac{5-3\sqrt{5}}{10} \\ c_2 = \frac{5+3\sqrt{5}}{10} \end{cases}$; ii) $\begin{cases} c_1 = -\frac{\sqrt{5}}{30} \\ c_2 = \frac{\sqrt{5}}{30} \end{cases}$.