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The Onset of a Darcy-Brinkman Convection using Nonequilibrium Model

by
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Abstract

A linearized analysis is performed in this paper in order to analyze the onset of Darcy-Brinkman convection in a fluid-saturated porous layer heated from below, by considering the case when the fluid and solid phases are not in local equilibrium. The problem is transformed into an eigenvalue equation which is solved in this paper by using an one-term Galerkin approach. Finally, an explicit relationship between the Darcy-Rayleigh number based on the fluid properties R and the horizontal wave number k is obtained. Minimization of R over k is performed analytically and finally, critical values for R and k are obtained for various values of the three parameters of the problem, namely the Darcy number, the porosity-scaled conductivity ratio and the scaled inter-phase heat transfer coefficient.

1 Introduction

The onset of convection in a porous layer heated from below is the "porous" version of the Benard problem in clear fluids. A critical review of the state-of-the art in this area of research was presented by Rees (2000). On the other hand, in a porous medium the volume averaged temperatures of the solid and fluid phases are generally different from one another and this is termed as local thermal non-equilibrium. Many studies in the literature of the non-equilibrium effects concentrated on the forced convective flows, but we are interested here in the combination of such effects with natural convective flows. A very recent review of the thermal non-equilibrium effects in natural convective flows was done by Rees and Pop (2005).
An earlier work in the field of thermal non-equilibrium effects on the onset of convection in a porous layer heated from below was the paper by Banu and Rees (2002), who were able to determine the Rayleigh number in a Darcy-like formulation. Further, Postelnicu and Rees (2003) extended the work by Banu and Rees (2002), by including the boundary effects as modelled by the Brinkmann terms. The included also the form-drag, but it was shown that these terms have no efect on stability criteria, since the basic state whose stability is being analysed was one of no flow.
The present paper extends the work by Postelnicu and Rees (2003), by taking into account isothermal rigid boundaries. In comparison with that work, where stress-free boundaries have been taken into account, here the analytical approach to find the critical Darcy-Rayleigh number and wavenumber at

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which the convection occurs is no more possible, so that we use in this paper a Galerkin approach. In a paper to come, Rees and Postelnicu (2005), we compare the results given by this Galerkin approach with those produced by more detailed numerical analysis of the resulting eigenvalue problem.

2 Analysis

We consider a porous layer saturated with an incompressible Newtonian fluid. The layer is heated from below, its lower surface being held at a temperature T_h, while the upper one is at a smaller temperature T_c . The porous layer has the depth d and is isotropic, but the local thermal equlibrium does not apply. The coordinates x and y are taken along the lower surface of the porous layer and normal to it, respectively.
The basic conduction profile, whose stability is studied here is

$$\psi = 0, \theta = \phi = 1 - y \tag{1}$$

where ψ is the dimensionless streamfunction, while θ and ϕ are temperature in the fluid and in the solid phase, respectively. We remark that y and in the next equations x, are dimensionless, normalized with the depth of the layer. The basic conduction profile (1) is perturbed by setting

$$\psi = \Psi, \theta = 1 - y + \Theta, \phi = 1 - y + \Phi \tag{2}$$

Considering that both form-drag and boundary effects are significant and invoking the Boussinesq approximation, the linearized perturbed equations in dimensionless form read

$$\partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 - D(\partial^4\Psi/\partial x^4 + 2\partial^4\Psi/\partial x^2\partial y^2 + \partial^4\Psi/\partial y^4) = R\partial\Theta/\partial x \tag{3}$$

$$\partial^2\Theta/\partial x^2 + \partial^2\Theta/\partial y^2 + \partial\Psi/\partial x + H(\Theta - \Phi) = 0 \tag{4}$$

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \gamma H(\Phi - \Theta) = 0 \tag{5}$$

In fact, these equations were derived in Postelnicu and Rees (2003). We only remark here that (3) is the equation of motion, while (4) and (5) are the energy equations in the fluid and solid phase, respectively. The dimensionless parameters appearing in equations (3-5) are: the Darcy-Rayleigh number R based on the fluid properties, the porosity-scaled conductivity ratio γ and the scaled inter-phase heat transfer coefficient H.
The boundary conditions for the perturbed equations (3-5) are

$$\Psi = \partial\Psi/\partial x = \Theta = \Phi = 0 \tag{6}$$

on $y = 0$ and $y = 1$. To this end, we remark that Postelnicu and Rees (2003) have solved the problem consisting of equations (3-5) but subjected to

$$\Psi = \partial^2\Psi/\partial x^2 = \Theta = \Phi = 0 \tag{7}$$

on $y = 0$ and $y = 1$, which means stress-free boundaries. In that case, we were able to get analytical solutions, supplementing our study with an asymptotic analysis for both small and large values of H. In the present case, of isothermal rigid boundaries, analytic progress is no more possible. Equations (3)-(5) admit solutions in the form

$$\Psi = f(y)sin(kx), \Theta = g(y)cos(kx), \Phi = h(y)cos(kx) \tag{8}$$

where k is the horizontal wavenumber. By substituting (8) into equations (3-5), we obtain

$$-D(f'''' - 2k^2 f'' + k^4 f) + (f'' - k^2 f) = -Rkg \tag{9}$$

$$g'' - (k^2 + H)g + kf + Hh = 0 \tag{10}$$

$$h'' - (k^2 + \gamma H)h + \gamma Hg = 0 \tag{11}$$

while the boundary conditions (6) become

$$f = f' = g = h = 0 \tag{12}$$

on y=0 and y=1.

3 Numerical analysis and results

We use a one-term Galerkin approach to solve our present problem. According to Finalyson (1972), the Galerkin technique has the advantage of dealing with many parameters very economically. We take

$$f = Af_1, f = Bg_1, h = Ch_1 \tag{13}$$

where f_1, g_1 and h_1 are trial functions and A, B and C are constants. The form of the boundary conditions (12) allow us to take

$$f_1 = y^4 - 2y^3 + y^2, g_1 = y(1 - y), h_1 = y(1 - y) \tag{14}$$

see also Pranesh et al. (2003), where the same boundary conditions were imposed. Multiplying (9) with f and integrating from 0 to 1, (10) with g and integrating from 0 to 1, and (11) with h and integrating from 0 to 1, we get after some algebra the following algebraic homogeneous system

$$-\left[\frac{4}{5}D + \frac{2}{105}(2k^2D + 1) + \frac{1}{630}k^2(1 + k^2D)\right] + \frac{1}{140}RkB = 0 \tag{15}$$

$$\frac{1}{140}kA - \left[\frac{1}{3} + \frac{1}{130}(k^2 + H)\right]B + \frac{1}{30}HC = 0 \tag{16}$$

$$\frac{1}{30}\gamma HB - \left[\frac{1}{3} + \frac{1}{130}(k^2 + \gamma H)\right] C = 0 \tag{17}$$

Equating the determinant of this system with zero gives an explicit expression for the Rayleigh number

$$R = \frac{a_{11}(a_{22}a_{33} - a_{23}a_{32})}{a_{12}^2 a_{33}} \tag{18}$$

where

$$a_{11} = -\frac{1}{630}Dk^4 - (\frac{4}{105}D)k^2 - \frac{4}{5}D - \frac{2}{105} \tag{19}$$

$$a_{12} = \frac{1}{140}k \tag{20}$$

$$a_{22} = -(\frac{1}{30}k^2 + \frac{1}{3} + \frac{1}{30}H) \tag{21}$$

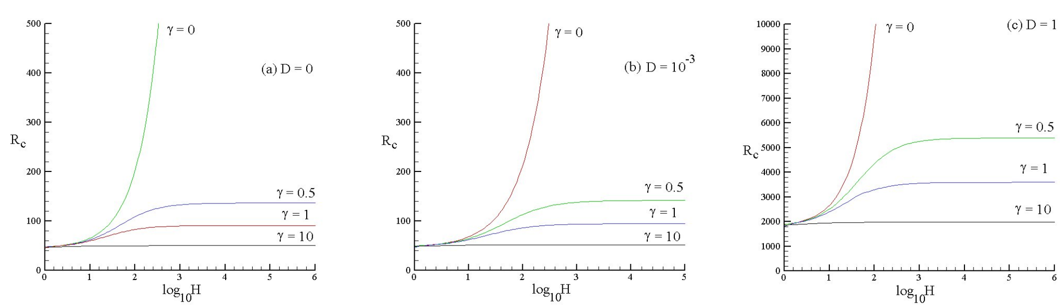
$$a_{23} = \frac{1}{30}H \tag{22}$$

$$a_{32} = \frac{1}{30}\gamma H \tag{23}$$

$$a_{33} = -(\frac{1}{30}k^2 + \frac{1}{3} + \frac{1}{30}\gamma H) \tag{24}$$

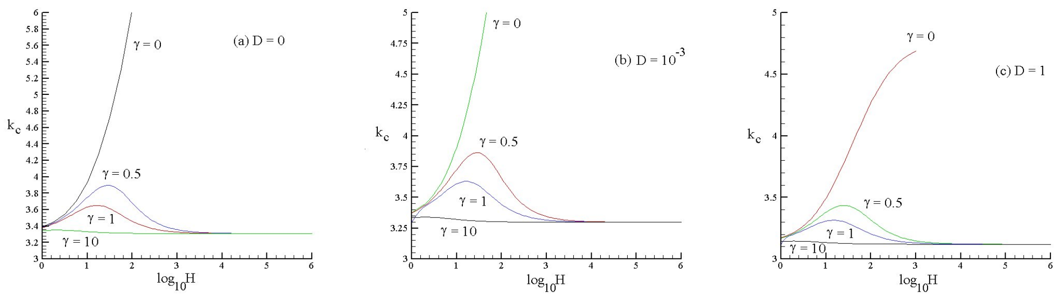
$$\tag{25}$$

Minimization of the Rayleigh number over k produces a polynomial equation, which can be solved by routine procedures.
In Figures 1 and 2 we present the behaviour of the critical Rayleigh number and critical wavenumber, respectively, as functions of H and γ for $D = 0$(Figs. 1a and 2a), $D = 10^{-3}$ (Figs. 1b and 2b) and $D = 10^{-3}$ (Figs. 1c and 2c).



(a) (b) (c)

Figure 1: Critical Rayleigh number as a function of H for various values of γ. a)$D = 0$, b)$D = 10^{-3}$, c)$D = 10^{-3}$.



(a) (b) (c)

Figure 2: Critical wavenumber as a function of H for various values of γ. a)$D = 0$, b)$D = 10^{-3}$, c)$D = 10^{-3}$.

References

1. Banu N, Rees D.A.S. 2002 Onset of Darcy-Benard convection using a thermal nonequilibrium model, International Journal of Heat and Mass Transfer, 45:2221-2228.

2. Finalyson, B.A. 1972 The method of weighted residuals and variational principles, Academic Press, New York.

3. Postelnicu A., Rees, D.A.S. 2003 The onset of Darcy-Brinkman convection in a porous medium using a thermal nonequilibrium model. Part 1: Stress-free boundaries, International Journal of Energy Research, 27:961-973.

4. Pranesh S. 2003 The effect of suction-injection-combination (SIC) on the onset of Rayleigh-Benard magnetoconvection in a micropolar fluid, International Journal of Engineering Science, 41:1741-1766.

5. Rees, D.A.S., Postelnicu, A. 2005 The onset of Darcy-Brinkman convection in a porous medium using a thermal nonequilibrium model. Part 2: Rigid boundaries, (in preparation).

6. Rees, D.A.S., Pop, I., 2005 Local thermal nonequilibrium in porous medium convection, Transport in Porous Media III, eds. D. B. Ingham and I.Pop (Pergamon) 147-173.

7. Rees, D.A.S. 2000 The stability of Darcy-Benard convection, Handbook of Porous Media ed. K.Vafai (Marcel Dekker) 521-558.