"Caius Iacob" Conference on Fluid Mechanics&Technical Applications **Bucharest, Romania, November 2005**

The evaluation of transient process in the sonic circuit of the high – pressure pipes used in line fuel injection systems for Diesel engines.

Abstract

The modern injection equipment produce a pollution level of the emissions which complies with the European Norms, a low fuel consumption level as well as a low level of noise of the Diesel engine. These antagonistic characteristics are achieved mainly by optimizing the burning process of the fuel in the burning chamber of the Diesel engine. Obtaining a mixture of optimal air/fuel ratio depends mainly on an adequate spraying of the fuel such as the drops are as small as possible as well as on the directioning of the pulverized fuel jets by the injector sprayer. For the conventional injection systems, the peak pressure is the most important measure for the quality of the forming the mixture in the burning chamber. Electro-hydraulic analogy as base of the sonic theory developed by the Romanian scientist George Constantinescu, leads to the possibility of modeling hydraulic systems by electric circuits through sonic resistances, capacities and inductivities. Electro-hydraulic modeling of the high-pressure pipe and injector allows evaluating the adapting condition for optimal adaptation of a chain of sonic quadripole. By considering the sonic injector circuit at injection phase and writing the transfer functions associated to the sonic quadruples, we are able to obtain the global transfer function, in its operational form. By solving the circuit we can obtain sonic potential differences and also sonic current in operational form. There are the expressions of pressure and deliveries differences in time range, after solving the Laplace transforms. Experimental results show the highest above the pressure peak at injector level, also its duration, the amplitude of the second peak and the attenuation in time domain of the pressure signal.

1. Electro-hydraulic modeling; the association of the hydraulic physica measures to the electrical physical measures; the Gogu Constantinescu formulas

Consider the equations of the rapid movements in time, of the fluids pipes under pressure, [1],

(S1)
$$\begin{cases} \frac{\rho}{A} \frac{\partial q}{\partial t} + \frac{\partial p}{\partial x} + R_u q = 0, \\ \frac{A}{\rho c^2} \frac{\partial p}{\partial t} + \frac{\partial q}{\partial x} = 0, \end{cases}$$
 (1)

¹ İng. Department Researcher Design, S. C. Mefin S. A. Sinaia, of Sinaia, România

² Prof. Dr. Ing., Department of Hydraulics and Hydraulic Machinery, University POLİTEHNİCA of Bucharest, România

where ρ is the density of the liquid, A – the current section of a transmission liquid column, q – flow, p – liquid pressure, c – propagating velocity of the perturbation in liquid columns and R_u – unit resistance coefficient in sonic transmissions.

Consider the equations of the long electric lines, [1], [4],

(S2)
$$\begin{cases} L_{l} \frac{\partial i}{\partial t} + \frac{\partial u}{\partial x} + R_{l} i = 0, \\ C_{l} \frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} + G_{l} u = 0, \end{cases}$$
(3)

where u = u(x, t) is the line voltage at the x distance from origin, i = i(x, t) is the line current at the x distance from origin, L_l - lineic inductivity, C_l - lineic capacity, R_l - lineic resistance and G_l - lineic conductance.

Formally comparing the equation systems (S1) and (S2) one can make an analogy between the physical measures from electricity and the corresponding ones from hydraulics. Systems (S1) and (S2) formally coincide if we consider $G_l = 0$ and in this case it is possible the achievement of the electro-hydraulic analogy.

We notice that in the case of the hydraulics circuits, the relation $G_l = 0$ is corresponding to the existence of a sonic transmission line without loss of liquid. Table 1 synthesizes the results of the formal comparison of the two equation systems, (S1) and (S2).

Table 1.

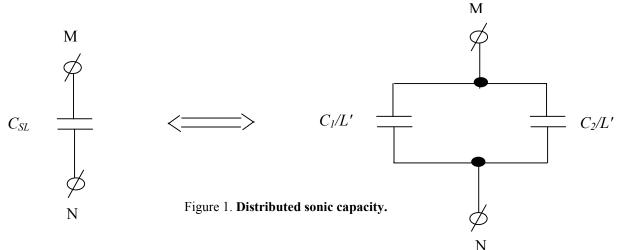
Electricity physical measures	i	и	L_l	C_l	R_l
Coresponding physical measures from hydraulics	q	p	ρ/Α	$A/\rho c^2$	R_u

2. High pressure pipe

The transmission of the mechanical power from the pump to the injector it's being done at sonic speed (i.e. the speed of sound in diesel oil), the transmission media being diesel oil considered as a compressible liquid.

The link between the pump and the injector is done by a high pressure pipe of length L'. The walls of the high pressure pipe are elastic and the diesel oil has also elastic properties. It results the existence of a distributed sonic capacities per unit length of a diesel gas pipe. We denote C_I the sonic capacity of the liquid column whose accumulation volume is V and with C_2 the sonic capacity due to the elasticity of the walls of the pipe. In figure 1 is presented the specific

capacities C_1/L' and C_2/L' , in parallel, and the equivalent sonic capacity corresponding to the unit length, C_{SL} .



The sonic capacities have the expressions, [1], [2],

$$C_1 = \frac{V}{E} = \frac{L'\Omega}{E},\tag{5}$$

$$C_2 = \frac{1,25}{E_1} \frac{D_m}{e} L'\Omega,$$
 (6)

where E is the elasticity module of oil diesel, L' – the length of the pipe, Ω – the section of the pipe, E_1 - the elasticity module of the material the pipe is made of, D_m – the average diameter of the pipe and e – the thickness of the walls of the pipe. We get

$$C_{SL} = \frac{C_1 + C_2}{L'} = \Omega \left(\frac{1}{E} + \frac{1,25}{E_1} \frac{D_m}{e} \right), \tag{7}$$

where C_{SL} is the distributed sonic capacity per unit length of the high pressure pipe. Sonic capacity C_2 is much smaller than the sonic capacity C_1 , approximately by 20 - 25 times, [2], so that we may consider

$$C_{SL} \approx \frac{\Omega}{E}$$
. (8)

The inertia of the liquid column determines a distributed sonic inductance per unit length, [1], [2], which may be written

$$L_{SL} = \frac{\rho_l}{g\Omega},\tag{9}$$

where ρ_l is the specific weight of the oil diesel.

Because of the friction between the interior walls of the high pressure pipe and the oil diesel we can assume the existence of a sonic resistance, [1], [2], distributed per unit length of the liquid column. The expression of the sonic resistance is

$$R_{SL} = K^* \frac{\rho_c}{g\Omega}, \tag{10}$$

where K^* is a constant whose value depends of the nature and the speed of the liquid, and ρ_c – the specific weight of the material the pipe is made of.

The high pressure pipe can be modeled by an infinite cascaded chain of elementary sonic quadruples, with concentrated constants R_{SL} , L_{SL} and C_{SL} . Assuming that the line is homogenous an elementary quadruple looks like in figure 2.

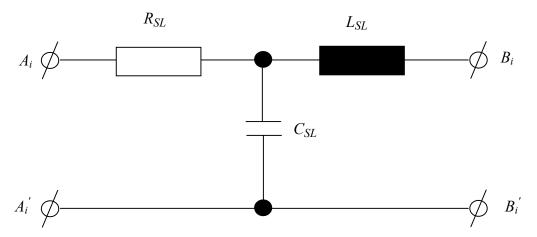
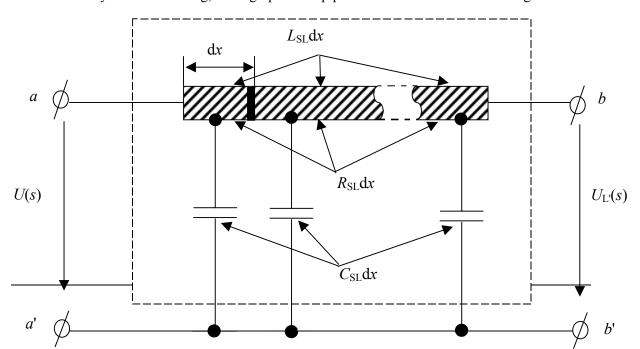


Figure 2. Electrical equivalent of a sonic circuit associated to an elementary quadruple with concentrated constants R_{SL} , L_{SL} and C_{SL} .

After electro-hydraulic modeling, the high pressure pipe can be assimilated with a long electrical



line.

The electrical equivalent of the sonic circuit associated to the high pressure pipe is represented figure 3.

Figure 3. The electrical equivalent of the sonic circuit associated to the high pressure pipe.

3. The transfer function associated to the sonic circuit of the high pressure pipe

We denote U(s) the sonic input voltage, and $U_L(s)$ the output sonic voltage of the high pressure pipe, in operational form, (figure 3). The transfer function associated with the sonic circuit of the high pressure pipe can be written

$$H_{lin}(s) = \frac{U_{L'}(s)}{U(s)}. \tag{11}$$

The expression of the sonic voltage in operational form in a transversal section of the high pressure pipe at the x distance from the sonic generator has the expression, [5], [6],

$$U(x,s) = U(s) \left\{ e^{-\gamma x} + \sum_{k=1}^{\infty} (-1)^k \rho_v^k \left[e^{-\gamma (2kL'+x)} - e^{-\gamma (2kL'-x)} \right] \right\}.$$
 (12)

If x = L', where L' is the length of the pipe, we get

$$U_{L'}(s) = U(s) \left\{ e^{-\gamma L'} + \sum_{k=1}^{\infty} (-1)^k \rho_{\nu}^k(s) \left[e^{-\gamma L'(2k+1)} - e^{-\gamma L'(2k-1)} \right] \right\},\,$$

and, finally, taking into account relation (11),

$$H_{lin}(s) = e^{-\gamma L'} + \sum_{k=1}^{\infty} (-1)^k \rho_v^k(s) \left[e^{-\gamma L'(2k+1)} - e^{-\gamma L'(2k-1)} \right]. \tag{14}$$

where γ stands for the propagation constant, and $\rho_{\nu}(s)$ is the operational reflexion coefficient. If we retain only the first two terms of the series (14), for k = 1, 2, we get

$$H_{lin}(s) \cong e^{-\gamma L'} + \left[\rho_{\nu}(s) \left(e^{-\gamma L'} - e^{-3\gamma L'} \right) + \rho_{\nu}^{2}(s) \left(e^{-5\gamma L'} - e^{-3\gamma L'} \right) \right],$$
 (15)

and if we retain only the first term, for k = 1,

$$H_{lin}(s) \cong e^{-\gamma L'} + \rho_{\nu}(s)e^{-\gamma L'} - \rho_{\nu}(s)e^{-3\gamma L'}$$
 (16)

4. The transfer function associated with the sonic circuit of the injector at the injection phase

We consider the equivalent of the sonic circuit of the injector initiating the injection, figure 4. We notice that the equivalent electric circuit of the sonic injector can be achieved exclusively with concentrated sonic elements, R_{ni} , L_i , C_i and R_{di} .

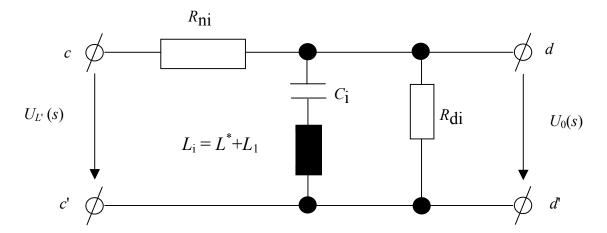


Figure 4. The electrical equivalent of the sonic circuit of the injector at the initiation of the injection phase.

Diesel gas leaks in case that the injector is closed due to the lack of fitness determine the existence of a theoretically infinite sonic resistance, R_{pi} . In this case the delay line represented by the high pressure pipe ends on infinite impedance. The needle of the sprayer the rod and the pressure spring build up a sonic circuit L_i - C_i oscillating series. The inertia the needle and the rod determines a sonic inductance L^* and the inertia of the weight of the spring determines a sonic inductance L_i . The portion between the needle of the injector and the nozzle introduces a sonic resistance, denoted R_{ni} . The sonic perditance of the nozzle orifices is denoted by S_{di} . These orifices have a constant flow section. The reverse of the sonic perditance S_{di} represents the sonic resistance of the nozzle orifices denoted R_{di} . We denote $U_L(s)$ the sonic voltage at the output of the high pressure pipe, in operational form, and with $U_0(s)$ the sonic voltage, in operational form, corresponding to the pressure of the diesel oil at the output of the nozzle's calibrated orifices of the sonic injector [3]. It has been denoted the operational argument with s. The transfer function associated with the sonic circuit of the injector has the form

$$H_{inj}(s) = \frac{U_0(s)}{U_{L'}(s)}. (17)$$

We determine the expression of the transfer function $H_{inj}(s)$ by making transformations of the circuit resulted due to the electro-hydraulic equivalence of the sonic circuit of the injector. Using the equivalent sonic impedances method, [4], we get at the first stage, the electrical equivalence of the sonic electrical circuit of the injector, the first equivalence, figure 5.

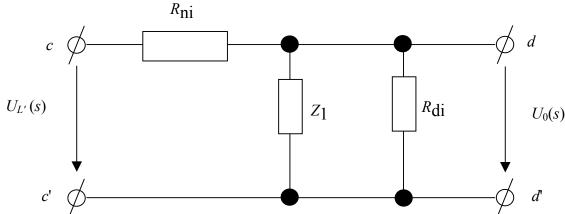


Figure 5. Electrical equivalent of the sonic circuit of the injector; first equivalence.

The equivalent sonic impedance Z_1 can be written, in operational form, [4],

$$Z_1(s) = s(L^* + L_1) + \frac{1}{sC_i} . {18}$$

If we consider the sonic impedance Z_{1di} , being equivalent to the sonic impedance Z_1 and R_{di} which are parallel, we obtain the electrical equivalent of the sonic circuit of the injector, the second equivalence, figure 6.

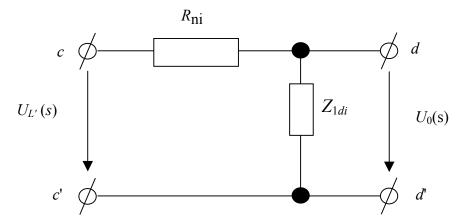


Figure 6. Electrical equivalent of the sonic circuit of the injector; the second equivalence.

The equivalent sonic impedance Z_{1di} can be written in operational form, [4],

$$Z_{1di}(s) = \frac{Z_1(s)R_{di}}{Z_1(s) + R_{di}} . {19}$$

Using the divisor formula, [4], we get

$$U_0(s) = \frac{Z_{1di}}{R_{ni} + Z_{1di}} U_{L'}(s), \tag{20}$$

out of which we get by using relation (17), the expression of the transfer function associated with the sonic circuit of the injector on the form

$$H_{inj}(s) = \frac{Z_{1di}}{R_{ni} + Z_{1di}} \ . \tag{21}$$

After successive transformations and taking into account that

$$L_i = L^* + L_1, (22)$$

we get

$$H_{inj}(s) = \frac{T_D s^2 + K_p}{T_1^* s^2 + T_2 s + 1} , \qquad (23)$$

where the following denominations have been made:

$$T_D = \frac{R_{di}}{R_{di} + R_{ni}} C_i L_i = K_p C_i L_i,$$
(24)

$$K_p = \frac{R_{di}}{R_{di} + R_{ni}} \,, \tag{25}$$

$$T_1^* = C_i L_i \,, \tag{26}$$

$$T_2 = \frac{R_{ni}R_{di}}{R_{di} + R_{ni}}C_i \ . \tag{27}$$

The series circuit C_i - $L_{i,j}$ figure 4., suggests the apparition of the oscillation phenomenon upon the needle of the sonic injector in the injection phase.

5. The transfer function explicitation $H_g(s)$ associated to the chain of sonic quadruples pipe-sonic injector

We consider the high pressure pipe and the sonic injector as a quadruples chain connected to the sonic generator in cascade, their interaction being made exclusively on the terminals, figure 7. The expression of the transfer function $H_g(s)$ can be written, [4],

$$H_g(s) = H_{lin}(s) \cdot H_{inj}(s). \tag{28}$$

SONIC GENERATOR

HIGH PRESSURE PIPE SONIC INJECTOR

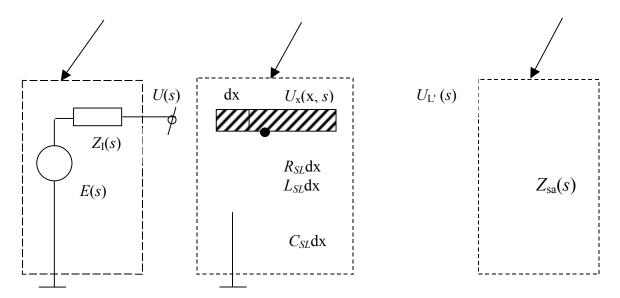


Figure 7. The electrical equivalent of the sonic circuit associated to a pumping section connected to the injector by a high pressure pipe.

Legend:

s =operational argument;

E(s) = the internal sonic voltage of the sonic generator, in operational form;

 $Z_{\rm I}(s)$ = the sonic impedance of the sonic generator, in operational form;

 R_{SL} , L_{SL} , C_{SL} , = the resistance, the inductance, and the sonic capacity distributed per unit length of the high pressure pipe;

 $Z_{\rm sa}(s)$ = the equivalent sonic impedance of the injector, in operational form;

U(s) = the sonic voltage at the input of the high pressure pipe, in operational form;

 $U_{\rm L}'(s)$ = the sonic voltage at the output of the high pressure pipe, in operational form;

 $U_x(x, s)$ = the sonic voltage in a transversal section of the high pressure pipe at the x distance from the sonic generator, in operational form.

Replacing the expression of $H_{lin}(s)$ and $H_{inj}(s)$ in relations (16) respectively (23) we obtain, for the injection phase,

$$H_g(s) = \left(e^{-\gamma L'} + \rho_v(s)e^{-\gamma L'} - \rho_v(s)e^{-3\gamma L'} \left(\frac{T_D s^2 + K_p}{T_1^* s^2 + T_2 s + 1}\right),\tag{29}$$

$$H_g(s) = \frac{U_0(s)}{U(s)}$$
 (30)

Considering the reverse Laplace transforms of the transfer functions $H_{inj}(s)$ and $H_{lin}(s)$ respectively,

$$h_{inj}(t) = L^{-1}\{H_{inj}(s)\},$$
 (31)

$$h_{lin}(t) = \mathbf{L}^{-1} \{ H_{lin}(s) \},$$
 (32)

We may write, [4],

$$H_g(s) = L\{h_{inj}(t) * h_{lin}(t)\},$$
(33)

in which the convolution product $h_{inj}(t)*h_{lin}(t)$ is given by relation, [4],

$$h_g(t) = h_{inj}(t) * h_{lin}(t) = \int_0^\infty h_{inj}(\tau) h_{lin}(t - \tau) d\tau , \qquad (34)$$

where

$$h_g(t) = L^{-1}\{ H_g(s) \},$$
 (35)

$$H_g(s) = L\{h_g(t)\} = \int_0^\infty h_g(t)e^{-st}dt$$
, (36)

s being the operational argument.

6. Obtaining the expression of the sonic voltage signal at the input of the high pressure pipe

A pumping section of a Diesel in line injection pump represents a sonic voltage impulse generator (pressure), figure 8. A pumping section (sonic generator) is composed by the injection cam, belaying-cleat with reel, the piston of the pumping element, the flow valve and the absorption valve, [3], [7]. The high pressure pipe represents a link element of the sonic circuit placed between the absorbing valve and the sonic injector.

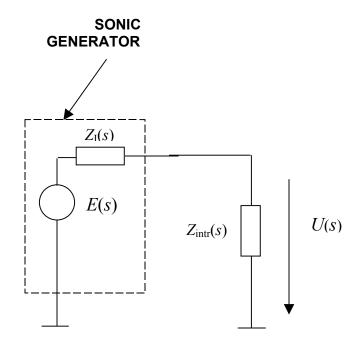


Figure 8. The electric equivalent of the sonic circuit of a pumping section, connected at a charge of $Z_{intr}(s)$ impedance.

The injecting of the fuel in the Diesel motor cylinder is a complex phenomenon of transmitting mechanical power from the injection pump to the injector by means of the sonic waves. The transmission media of the sonic waves is the diesel oil found in the pressure sonic generator, in the high pressure pipe and in the injector. The internal sonic impedance of the sonic generator is written in operational form as a function of the pressure sonic generator impedances $Z_I^*(s)$, of the flow valve $Z_{sd}(s)$ and of the absorption valve $Z_a(s)$,

$$Z_I(s) = Z_I^*(s) + Z_{sd}(s) + Z_a(s).$$
(37)

The level of the signal at the input of the line according with the divisor formula, [4], (figure 8), is given by the sonic voltage

$$U(s) = \frac{Z_{\text{int}\,r}(s)}{Z_I^*(s) + Z_{sd}(s) + Z_a(s) + Z_{\text{int}\,r}(s)} E(s),$$
(38)

where $Z_{\text{int}r}(s)$ is the sonic impedance, in operational form, seen by the sonic generator, [4],

$$Z_{\text{int }r}(s) = \frac{Z_{sa}(s)ch\gamma L' + Z_{0}(s)sh\gamma L'}{Z_{sa}(s)sh\gamma L' + Z_{0}(s)ch\gamma L'} Z_{0}(s),$$
(39)

where Z_0 (s) is the operational characteristic sonic impedance of the line (corresponding to the high pressure pipe), and $Z_{sa}(s)$ is the operational sonic impedance of the injector. The operational reflection coefficient,

 $\rho_{v}(s)$ has the expression, [4],

$$\rho_{\mathcal{V}}(s) = \frac{Z_{sa}(s) - Z_{0}(s)}{Z_{sa}(s) + Z_{0}(s)} \ . \tag{40}$$

If we neglect the leaks of Diesel oil, the propagation constant of the delaying line represented by the high pressure pipe is written

$$\gamma = \gamma(s) = \sqrt{(R_{SL} + L_{SL})sC_{SL}} , \qquad (41)$$

Replacing R_{SL} , L_{SL} and C_{SL} , out of the relations (8), (9) and (10), we get

$$\gamma = \gamma(s) = \sqrt{\frac{(K^* \rho_c + s\rho_l)s}{gE}} . \tag{42}$$

For a lossless line without leakage ($G_{SL} = 0$ and $R_{SL} = 0$ respectively), the characteristic impedance, in operational form, has the expression

$$Z_0(s) = \sqrt{\frac{R_{SL} + sL_{SL}}{G_{SL} + sC_{SL}}} = \sqrt{\frac{L_{SL}}{C_{SL}}} = \frac{1}{\Omega} \sqrt{\frac{\rho_l E}{g}}.$$
 (43)

The impedance of the charge can be written taking into account the electrical equivalent of the sonic circuit of the injector, figure 5, under the form

$$Z_{sa}(s) = R_{ni} + \frac{R_{di}\left(sL_i + \frac{1}{sC_i}\right)}{R_{di} + sL_i + \frac{1}{sC_i}} = R_{ni} + \frac{R_{di}\left(1 + s^2L_iC_i\right)}{1 + sC_iR_{di} + s^2C_iL_i}.$$
(44)

The case of in line Diesel injection systems is typical for transmitting of relatively great forces in a small amount of the Δt 2 ms, by means of the Diesel oil in the high pressure pipe to a sonic receptor placed at a distance from the sonic generator. Because the liquid is elastic and has a finite mass the transmission is not instantaneous but depends by the speed of sound in Diesel oil. The own liquid column frequency from the high pressure pipe is in general several times greater than the frequency by which injections take place. The pressure waves have the time to travel the pipe several times during two successive injections, being reflected at the sonic injector's end as well as at the coupling end of the pumping section. The relexion coefficient ρ_v , can have positive or negative values. In order for the line to be adapted with charge it's necessary that $\rho_v \approx 0$. In this case the sonic receptor (the injector) absorbs almost the entire energy of the direct wave.

7. Experimental results

The first two experiments refers at the relation of the pressure as a function of time in the vicinity of the coupling where x = 0. Thus the instantaneous pressure it's measuring at the entry of the high pressure pipe, for the revolution n = 1300 rpm corresponding to the rated power (figure 9), as well as for the revolution n = 800 rpm, corresponding to the maximum torque (figure 10).

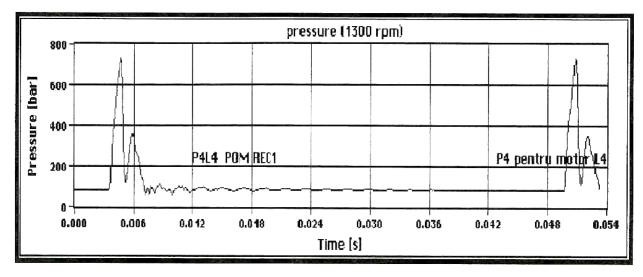


Figure 9. The pressure diagram at the entry in the high pressure pipe, for the rated power revolution, n = 1300 rpm.

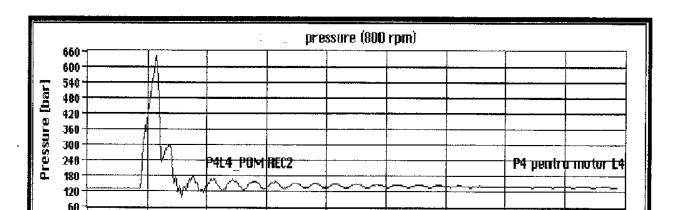


Figure 10 The pressure diagram measure	ed at the entry in the high pressure pipe for maximum torque			
rigure 10. The pressure diagram measure	revolution, $n = 800 \text{ rpm}$.			
The following measurements have been made using an outlet placed at the high pressure				
pipe's end towards the injector, where $x = L'$, figures 11 and 12.				
L				

Figure 11. The pressure diagram measured at the exit from the high pressure pipe, for an Φ =10 mm element.

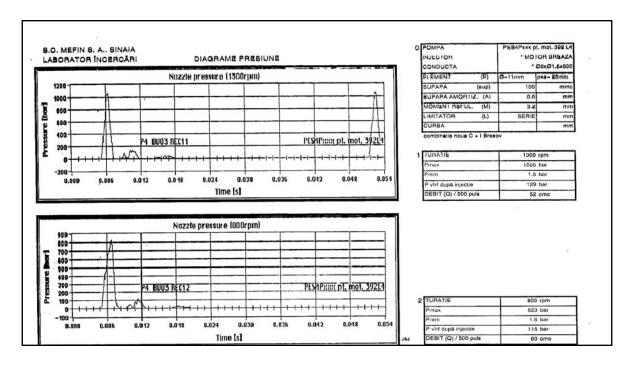


Figure 12. The pressure diagram measured at the exit from the high pressure pipe, for an Φ =11 mm element.

The following optimizations criteria have been considered:

- C1) P_{max} , representing the maximum pressure (the peak of pressure), has to have the amplitude as big as possible;
- C2) $P_{\text{varf după injecție}}$, representing the amplitude of the pressure corresponding to the first peak after injection, has to be as small as possible;
- C3) P_{rem} , representing the remnant pressure, has to be as small as possible, even 0 bars. Correspondingly, the high pressure pipes that have been used had the following geometrical dimensions:
- 1) Ø 6 x Ø 1,5 x 600 (element Ø 10);
- 2) Ø 6 x Ø 1,75 x 600 (element Ø 10);
- 3) Ø 6 x Ø 1,5 x 800 (element Ø 11);
- 4) Ø 6 x Ø 1,5 x 600 (element Ø 11);

The optimization criteria led to the final choice: $\emptyset_{\text{ext}} = 6 \text{ mm}$, $\emptyset_{\text{int}} = 1,5 \text{ mm}$ and L = 600 mm. At the high pressure pipe's end towards the injector, the first pressure peak after the main injection has a value around $P_{\text{varf după injecție}} \approx 120 \text{ bar}$, inferior to the pressure needed to open the injector, $P_{\text{inj}} = 250 \text{ bar}$. That is the reason why, for a pumping section corresponding to the P sized in line pump we don't get post injection phenomenon. These implementations achieve a very low remnant pressure and a very good adjustment with the sonic injector. Due to the high values of the first peak of pressure, a very good atomization of the fuel in the injection phase is achieved. The first implementation achieves a pressure peak of $p_{v1} = 847 \text{ bar}$ at $n_1 = 1300 \text{ rpm}$ and $p_{v2} = 765 \text{ bar}$ at $n_2 = 800 \text{ rpm}$ respectively. The second implementation achieves a peak pressure of $p_{v1}^* = 1066 \text{ bar}$ at $n_1 = 1300 \text{ rpm}$ and of $p_{v2}^* = 823 \text{ bar}$ at $n_2 = 800 \text{ rpm}$.

Bibliography:

- [1] Cioc D., Măruță Al., Bazele teoretice ale sonicității, I. C. București, 1972.
- [2] C o n s t a n t i n e s c u G o g u, Teoria sonicității, Editura Academiei, București, 1985.
- [3] D u m i t r u P e t r e, *Pompe de injecție și injectoare pentru automobile și tractoare*, Editura Ceres, București,1988.

- [4] M o c a n u C. I., *Teoria circuitelor electrice*, Editura Didactică și Pedagogică, București, 1979.
- [5] Panaites cu Valeriu, Pîslaru Dănes cu Lucian, Electro-hydraulic modeling of the fuel injection systems, based on the George Constantinescu's sonic principle of energy transmision by vibrations, The 10th National Jubilee Conference, S.P.E.R.I.N., University Politehnica of Bucharest, 30 September, 2000.
- [6] Pîslaru Dănescu Lucian, Panaitescu Valeriu, Development of fuel injection systems by using sonic theory, Conferința Națională "Caius Iacob" de Mecanica fluidelor și aplicațiile ei tehnice, Universitatea Tehnică Gh. Asachi, Iași, 21 22 Septembrie 2000. Buletinul Institutului Politehnic din Iasi, Universitatea Tehnica "Gh Asachi" din Iasi, Tomul XLVI(L), Seria Constructii de masini, pp. 263-266.
- [7] Turcoiu T., Boncoi I., Time A., Echipamente de injecție pentru motoarele cu ardere internă, Editura Tehnică, București, 1987.