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Stability of Particular Immiscible Flow in Porous Media

by
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Abstract

We consider the saturation model for immiscible flow in porous media. We prove that a particular basic wave solution is stable. In contrast, the planar interface in immiscible Hele-Shaw flow is always unstable. Our stability system contains some terms negelcted by prevoius authors. The result is obtained by direct integration of the stability problem.

We consider the saturation model in the presence of capillary pressure. An infinite plane homogeneous porous medium is saturated with two immiscible phases: water and oil. The equations for this porous media flow are given by

$$\phi\, \partial S_i/\partial T + \nabla \cdot v_i = 0, \tag{1}$$

$$v_i = -K\, \frac{k_i}{\eta_i}\, \nabla P_i, \tag{2}$$

$$\nabla \cdot v_T = 0, \tag{3}$$

$$P_o - P_w = P_c, \quad S_o + S_w = 1, \tag{4}$$

where the subscripts $i = w, o$ denote displacing (water) and displaced (oil) phases, ϕ is the constant porosity. The capillary pressure P_c is given using the Leverett function. We consider the notations

$$a(S) = k[\frac{k_0}{\eta_0} + \frac{k_w}{\eta_w}], \quad c(S) = k[\frac{k_0}{\eta_0}], \quad b(S) = k[\frac{k_w}{\eta_w}]\frac{dP_c}{dS}. \quad S = S_w, \quad P = P_0$$

The initial and boundary conditions are:

$$S(x, y, 0) = S_i(x, y) = \left\{ \begin{array}{ll} S_l, & for \quad x = 0 \\ S_r, & for \quad x \to \infty \end{array} \right\} \tag{5}$$

$$S(0, y, T) = S_l; \quad S(\infty, y, T) = S_r. \tag{6}$$

We study the following one-dimensional traveling wave solution profiles of the above system

$$S(x, T) = \overline{S}(\chi), \qquad \chi = (x - Ut),\ t = T, \tag{7}$$

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$$\overline{S}(\chi) \to S_r \text{ as } \chi \to \infty, \quad \overline{S}(\chi) \to S_l > S_r \text{ as } \chi \to -\infty, \quad \frac{d\overline{S}}{d\chi} \to 0 \text{ as } \chi \to \mp\infty. \tag{8}$$

where U is a "front" velocity given in terms of S_l, S_r. Our system admits the basic traveling solution

$$c(\overline{S}) f_w(\overline{S}) \frac{dP_c}{dS} \frac{d\overline{S}}{d\chi} = U\,\phi \overline{S} - v_T f_w(\overline{S}) + A, \tag{9}$$

$$c(\overline{S}) \frac{d\overline{P}}{d\chi} = U\,\phi \overline{S} + C, \tag{10}$$

where C is a constant and f_w is the fractional flow function.
We study the linear stability of the solution $(\overline{S}, \overline{P})$. We consider the perturbed solutions and obtain the following system:

$$S(x,y,t) = \overline{S}(\chi) + \epsilon s_1, \quad P(x,y,t) = \overline{P}(\chi) + \epsilon p_1, \quad (s_1, p_1) = (s(\chi), p(\chi))\, e^{(iky+\sigma t)}, \tag{11}$$

$$a(\overline{S} + \epsilon s_1) = a(\overline{S}) + a_s(\overline{S})\epsilon s_1 + O(\epsilon^2), \tag{12}$$

$$p''\,\overline{c} + p'\,c'(\overline{S}) - \overline{c}\,k^2 p + \sigma\,\phi\,s - \phi U s' + \frac{d}{d\chi}\left(c_s(\overline{S})\,s\,\frac{d\overline{P}}{d\chi}\right) = 0, \tag{13}$$

$$p''\,\overline{a} + p' a'(\overline{S}) - \overline{a}\,k^2 p - s''\,\overline{b} - b'(\overline{S})\,s' + \overline{b}\,k^2\,s + \\ + \frac{d}{d\chi}\left(a_s(\overline{S})\frac{d\overline{P}}{d\chi}s - b_s(\overline{S})\frac{d\overline{S}}{d\chi}s\right) = 0, \tag{14}$$

where $' = d/d\chi$ and $\overline{a}, \overline{b}, \overline{c}$ are the values of a, b, c for the traveling wave $\overline{S}$. These equations are similar with ones studied in [1] except the terms containing $a_s(\overline{S}), b_s(\overline{S}), c_s(\overline{S})$.
The above system is simplified for a particular case of relative permeability k_w: $\frac{dk_w}{dS}(S_r) = k_w(S_r) = 0$. Moreover, for large χ we can consider $a' = b' = c' = 0$. By a straight calculus, we obtain only one relation for the unknown eigenfunction $s(\chi)$:

$$b(S_r)\,c(S_r)\,(s'' - k^2 s) + \sigma\,\phi\,a(S_r)\,s - a(S_r)\phi U s' + \\ s'\,\frac{d\overline{P}}{d\chi}\,(a(S_r)\,c_s(S_r) - c(S_r)\,a_s(S_r)) = 0. \tag{15}$$

The last term in (15) is zero, using the properties of k_w and we have

$$-\alpha\,s'' + \phi U s' + \alpha k^2 s = \omega\,s \tag{16}$$

$$s(\infty) = 0. \tag{17}$$

$$\alpha = b(S_r)\,c(S_r)/a(S_r) < 0, \ \omega = \sigma\,\phi, \ 0 < \phi < 1 \tag{18}$$

As in [1], the modulus of the eigenfunction s admits a maximum value. Therefore we consider in (16) $s(\chi) = exp(-r\chi)cos(q\chi), \quad r > 0$. For this type of "eigenfumctions" we obtain $\omega < 0$, then our basic solution (9)-(10) is stable.

References

[1] Y. C. YORTSOS AND F. J. HICKERNELL, *Linear stability of immiscible displacement in porous media*, SIAM J. Appl. Math., 49(3), (1989), pp. 730-748.