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Mathematical Models and Optimizations of Naval Sail Systems

by
Mircea LUPU[1] , Ştefan NEDELCU [2] and Adrian POSTELNICU[3]

Abstract

The paper presents mathematical models and methods for the optimization of wind propelled sail profiles. In order to solve limit issues, direct or inverse methods have been used. Both cases of wind circulation around the sail profile and circulation-free cases have been aproached. For sail optimization purposes, flaps sails are considered assimilated to a point-vortex.

Key words and phrase: Direct and inverse problem; Jensen's inequality; current tubes methods; optimization problem

Mathematics Subject Classification: 65M32, 65R20, 76G25

1 Introduction

In this paper aerodynamique profiles theory we will use in order to solve and optimization mathematical models for naval sail systems. We consider rigide sails, plates rights or curves and search optimal shape for maximal propulsion in two limit cases, wind rectangular on the plate and wind paralel with profile chord. For optimization of performances, we study flaps sail model. These researches have start point theoretical and practical experiments of Naval Academy "Mircea cel Bătrân" team and the Reserch Institut for Wind Energy from Braşov. We present now theoretical model for plate sail used the hidrodynamic potential theory with free surfaces. The stationary potential plane flow of an inviscid fluid is considered in the absence of mass forces. Relating the velocity field $\vec{v} = u\vec{i} + v\vec{j}, u = u(x,y), v = v(x,y)$, to the frame in the physical flow domain $D_z, z = x + iy$, then within the hypothesis as well as from the continuity equation $div\vec{v} = 0$ and the condition for an irrotational flow ($rot\vec{v} = 0$) [3] [9],we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \tag{1}$$

In this case, $\vec{v} = grad\varphi(x,y)$, and the velocity potential $\varphi = \varphi(x,y)$ is a harmonic function, $\Delta\varphi = 0$ in D_z. By introducing the stream function $\psi = \psi(x,y)$, the harmonic conjugate of φ in D_z, $\Delta\psi = 0$

[1]Transilvania University of Brasov, Bdul Eroilor, No 29, Brasov, 500036
E-mail: m.lupu@info.unitbv.ro
[2]Air Forces Academy "Henry Coanda" Brasov, Romania
E-mail: stefan.nedelcu@xnet.ro
[3]Transilvania University of Brasov, Bdul Eroilor, No 29, Brasov, 500036
E-mail: adip@unitbv.ro

in D_z, the relations (2) are obtained:

$$u = \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}, v = \frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{2}$$

Thus, the complex potential of the flow is considered to be $f = f(z)$ and the complex velocity$w = w(z)$:

$$f(z) = \varphi x, y + i\phi(x, y), \bar{w} = \frac{df}{dz} = u - iv \tag{3}$$

In the hodograph plane (V, θ), where $V = \sqrt{(}u^2 + v^2)$ is the magnitude of the velocity and $\theta = argw = arctg\frac{v}{u}$ the complex velocity angle with $x'Ox$ axis, the following relations can be written:

$$W = V + i\theta, u = V\cos\theta, v = V\sin\theta, w = Ve^{i\theta} \tag{4}$$

With (4), the transition relation $f = f(z)$ is obtained:

$$d\varphi + id\psi = (u - iv)(dx + idy) \tag{5}$$

In the case of free surface flow, the domain D_z is generally bounded by polygonal rigid walls, curvilinear obstacles and stream lines detaching from walls or obstacles. Along these free lines the velocity, the pressure and the density are $V^0, p^0, \rho = \rho^0 = const$. Applying Bernoulli's law for incompressible fluids ($\rho = const$) along a streamline [3], [9], $\psi = const$, we have

$$\frac{1}{2}V^2 + \frac{p}{\rho} = \frac{1}{2}V^{0^2} + \frac{p^0}{\rho} \tag{6}$$

In the hypothesis (Hyp), we generally consider the plane parallel to infinity flow of an inviscid fluid which encounters a curvilinear obstacle $\lim_{|z|\to\infty} \vec{v} = V^0\vec{i}$. The Ox axis is the symmetry axis. This is the "Helmholtz model" of the symmetrical obstacle in unlimited fluid. The repose zone is downstream and is delimited by obstacle and free lines. In the case of a curvilinear domain D_z it is generally difficult to obtain directly $f = f(z)$ and $w = w(z)$ by solving the boundary problem, therefore it should be introduced a canonic auxiliary domain $D_\zeta = \xi + i\eta$. In this paper, we choose the half-plane $D_\zeta^+ = \xi + i\eta, \eta > 0$, as canonic domain, we give some theoretic and applied results and we emphasize the computation techniques for analytic functions or nonlinear operators. We try to determine the analytic function $f = f(\zeta)$ which is the conformal mapping $D_f^+ \longleftrightarrow D_\zeta^+$, with

$$f(\bar{\zeta}) = 0, \varphi_\xi = \psi_\eta, \varphi_\eta = -\psi_\xi \tag{7}$$

In order to obtain the analyticity conditions for the velocity $W(V, \theta)$ in D_ζ^+, we introduce the Jukovski's function ω [6], by considering $V = V^0$ along the free lines:

$$\omega = t + i\theta, \bar{w} = V^0 e^{-\omega}, t = \ln\frac{V^0}{V}, 0 \le V \le V^0 \tag{8}$$

$$\theta_\psi = t_\varphi, \theta_\varphi = -t_\psi, \varphi_\theta = -\psi_t, \varphi_t = \psi_\theta, \omega_{\bar{f}} = 0, f_{\bar{\omega}} = 0 \tag{9}$$

Now, we consider the following theorems.

Theorem **1** *In the hypothesis* (Hyp)*, if there is a conformal mapping* $f = f(\zeta)$*,* $f_{\bar{\zeta}} = 0$*, with* $D_f^+ \longleftrightarrow D_\zeta^+$*, then* $z = z(\zeta)$ *is analytic with* $D_z^+ \longleftrightarrow D_\zeta^+$*.*

Theorem **2** *In the hypothesis* (Hyp)*, if the function* f *is analytic in* ζ *and realizes a conformal mapping* $D_f^+ \longleftrightarrow D_\zeta^+$*, then* $\omega = \omega(\zeta)$ *is analytic and it is the conformal mapping* $D_\omega^+ \longleftrightarrow D_\zeta^+$*.*

Writing the relation (5) along a stream line $\psi = const$ and using Theorem 1 one obtains the equations of this stream line (obstacle, free lines) and with $\eta = 0$ $\frac{\partial\varphi}{\partial\eta}\,|_{\eta=0} = 0$, we have

$$x(\xi) = \int_{\xi_0}^{\xi} \varphi_\xi \frac{\cos\theta}{V} d\xi + x_0, y(\xi) = \int_{\xi_0}^{\xi} \varphi_\xi \frac{\cos\theta}{V} d\xi + y_0 \tag{10}$$

In order to obtain the functions $V = V(\xi)$ and $\theta = \theta(\xi)$, we carry out Theorem 2 and solve a mixed Riemann-Hilbert or Volterra problem for $\omega = \omega(\zeta)$, [2], [9]. By (8), we then obtain $w = w(\zeta)$. Thus, the movement $f = f(z), w = w(z)$ (or parametric $f = f(\zeta), w = w(\zeta)$) is obtained by the composition $D_f^+, D_z^+, D_\omega^+ \longleftrightarrow D_\zeta^+$. Next we realize Theorems 1 and 2 for the "Helmholtz model" obtaining the integral singular equations for direct and inverse problems too. So, in the conditions stated above, we consider the plane flow of an unlimited fluid, moving infinitely upstream in an uniform translation movement of velocity $\vec{V^0} = V^0\vec{i}$. The fluid hits a symmetrical curvilinear obstacle (BOB') and in the points B and B', the free streamlines (BC) and $(B'C')$ of V^0 velocity are detached. The $x'Ox$ axis is the symmetry axis A_0O. The point A_0 is at infinity upstream, $V(\vec{A_0}) = V^0\vec{i}$. The free lines are asymptotically parallels to $x'Ox$ axis, $V(\vec{C}) = V(\vec{C'}) = V^0\vec{i}$. $(CBOB'C')$ will be the repose zone behind the obstacle and $V(\vec{O}) = 0$ We consider the correspondence between the domains D_z^+, D_f^+, D_w^+

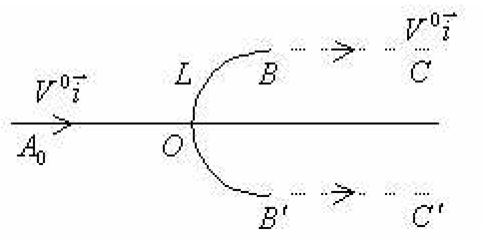

Fig.1

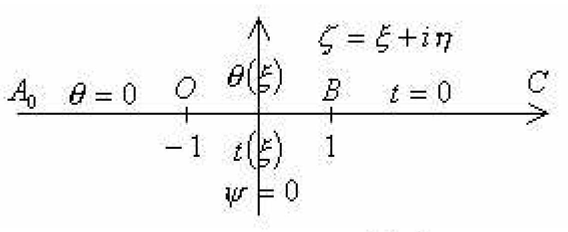

Fig.2

with the half-plane $D_\zeta^+, \eta > 0$, so that the boundary (A_0OBC) will be replaced by the $\eta = 0$ axis, $\xi \in (-\infty, \infty) : A_0(-\infty), O(-1).B(1), C(+\infty)$ (Fig.2). So, the obstacle (OB) is the segment $(-1, 1)$ and, in the physical plane D_z, the length of (OB) is L.

2 The deduction of the integral equations for plate sails with rectangular wind on plate

In order to determine the complex potential $f = \varphi + i\psi$ in the half-plane $D_\zeta^+, \eta > 0$, we have to solve the following Dirichlet problem: find an analytic function $f(\zeta) = \varphi + i\psi$ in $\eta > 0$ such that $\psi = 0$ on

$\eta = 0, \xi \in (-\infty, \infty)$. The solution of this problem is:

$$f(\zeta) = A\zeta, A > 0, \frac{\partial \varphi}{\partial \xi} |_{\eta=0} = A \, , \frac{\partial \varphi}{\partial \eta} |_{\eta=0} = 0 \tag{11}$$

We determine $\omega = \omega(\zeta)$ in two manners. We have to find the analytic function $\omega = \omega(\zeta = t + i\theta$ in $\eta > 0$ knowing the following values on the boundary $\eta = 0 : \theta = 0, \xi \in (-\infty, -1), \theta = \theta(\xi)$ or $t = t(\xi), \xi \in (-1, 1); t = 0, \xi \in (1, \infty)$. These are mixed problems and we transform them into Dirichlet problems for the analytic functions S_1, S_2 in $\eta > 0$:

$$S_1(\zeta) = R_1 + iI_1 = \frac{\omega(\zeta)}{\sqrt{(\zeta + 1)}} :$$

$$R_1 = 0, \xi \in (-\infty, -1) \cup (1, +\infty); R_1 = \frac{t(\xi)}{\sqrt{\xi + 1}}, \xi \in (-1, 1)$$

$$S_2(\zeta) = R_2 + iI_2 = \frac{\omega(\zeta)}{\sqrt{(\zeta - 1)}} :$$

$$R_1 = 0, \xi \in (-\infty, -1) \cup (1, +\infty); R_1 = \frac{t(\xi)}{\sqrt{1 - \xi}}, \xi \in (-1, 1)$$

From the Cisotti formula we have

$$\omega(\zeta) = \frac{\sqrt{\zeta + 1}}{\pi i} \int_{-1}^{1} \frac{t(s)}{\sqrt{s + 1}} \frac{ds}{s - \zeta} + C_1 i, \zeta \in D_\zeta^+ \tag{12}$$

or

$$\omega(\zeta) = \frac{\sqrt{\zeta - 1}}{\pi i} \int_{-1}^{1} \frac{\theta(s)}{\sqrt{1 - s}} \frac{ds}{s - \zeta} + C_2 i, \zeta \in D_\zeta^+ \tag{13}$$

with the constants $C_1 = 0, C_2 = 0$ if $V(\xi = -1) = 0$ and $t(\xi = -1) = +\infty$. Applying the Sohotski-Plemelj formula [2],[3] to the Cauchy integrals (12), (13) when $\zeta = \xi, \eta = 0^+$ for $\xi \in (-1, 1)$ we obtain

$$\theta(\xi) = \frac{\sqrt{\zeta + 1}}{\pi} \int_{-1}^{1} \frac{t(s)}{\sqrt{s + 1}} \frac{ds}{s - \zeta} \zeta \in (-1, 1) \tag{14}$$

$$t(\xi) = \frac{\sqrt{\zeta - 1}}{\pi} \int_{-1}^{1} \frac{\theta(s)}{\sqrt{1 - s}} \frac{ds}{s - \zeta}, \zeta \in (-1, 1) \tag{15}$$

The singular integrals are taken in the sense of Cauchy's principal value. By (5), the arc element on (OB) is

$$dS = \varphi_\xi \frac{d\xi}{V(\xi)}, S = \int_{-1}^{\xi} A \frac{d\xi}{V(\xi)}, \xi \in (-1, 1) \tag{16}$$

and from (16) and (8) the length (OB) is

$$L = A \int_{-1}^{1} \frac{d\xi}{V(\xi)} = \frac{A}{V^0} \int_{-1}^{1} e^{t(s)} ds. \tag{17}$$

If the length L and the distribution of the velocity along (OB) are given, then the parameter A can be found. With these results we can emphasize the inverse problems for the "Helmholtz model" with curvilinear obstacle. If the distribution of the velocity $V = V(\theta)$, i.e. $t = t(\theta(\xi))$, or the pressure $p = p(\theta)$ on the profile (OB) are given, then (15) is a singular integral equation with the unknown function $\theta = \theta(\xi)$. Next, the functions $\omega = \omega(\zeta), w = w(\zeta)$ may be deduced (see (12), (13)). The relations (10) give us the equation of the profile (OB): $z = z(\xi) = \int_{\xi_0}^{\xi} \varphi_\xi \frac{e^{i\theta}}{V} d\xi + z_0$. We remark that the relations (14), (15) are inversion formulae for $\theta(\xi) \longleftrightarrow t(\xi), \xi \in (-1, 1)$.

2.1 The problem of the normal plate in unlimited flow

Let us consider in the same hypothesis the plane flow of an unlimited fluid, moving infinitely upstream in an uniform translation movement of velocity $\vec{V^0} = V^0 \vec{i}$ which encounters the symmetric plate (BOB'). Similarly, the free streamlines (BC) and $(B'C')$ are detached in the points B and B'. This is the Helmholtz's problem. We will solve this direct problem by means of the results of Sections 2 and 3 prescribing along the plate $\theta(\xi) = \frac{\pi}{2}, \xi \in (-1, 1)$. Due to the symmetry, we consider the physical half-plane $D_z^+, y > 0$, delimited by (A_0OBC) with the same correspondence in the half-plane $D_\zeta^+, \eta > 0$. Thus, replacing $\theta(\xi) = \frac{\pi}{2}$ in (15) we get the distribution of the velocity on the plate (OB)

$$t(\xi) = \frac{1}{2} \ln \frac{\sqrt{2} + \sqrt{1-\xi}}{\sqrt{2} - \sqrt{1-\xi}}, V(\xi) = V^0 \left(\frac{\sqrt{2} - \sqrt{1-\xi}}{\sqrt{2} + \sqrt{1-\xi}} \right)^{\frac{1}{2}}, \xi \in (-1, 1) \tag{18}$$

It's clear that $V(O) = V(\xi = -1) = 0$ and $V(B) = V(\xi = 1) = V^0$. Computing length of the plate (BOB') and using the relations (18), (17) we have $L = \frac{A}{V^0} \frac{\pi+4}{2}, A = \frac{2LV^0}{\pi+4}$. Knowing F and V^0one determines the parameter A. To compute the distribution of the velocity along (A_0O) with $\theta = 0$, we remake the computations of Section 3 with distribution (18) and we get

$$t(\xi) = \frac{\sqrt{-1-\xi}}{\pi} \int_{-1}^{1} \frac{t(s)}{\sqrt{1+s}} \frac{ds}{s-\xi} = \frac{1}{2} \ln \frac{\sqrt{1-\xi} + \sqrt{2}}{\sqrt{1-\xi} - \sqrt{2}}, \xi \in (-\infty, -1) \tag{19}$$

$$V = V^0 \left(\frac{\sqrt{1-\xi} - \sqrt{2}}{\sqrt{1-\xi} + \sqrt{2}} \right)^{\frac{1}{2}}, \xi \in (-\infty, -1) \tag{20}$$

$$P = \frac{\rho {V^0}^2}{2} \int_{-1}^{1} \left[1 - \left(\frac{V}{V^0} \right)^2 \right] \frac{A}{V(\xi)} d\xi = \frac{\rho {V^0}^2}{2} A \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} d\xi = \pi A \frac{\rho {V^0}^2}{2} \tag{21}$$

Then using (19), the drag coefficient is given by

$$C_x^P = \frac{2\pi}{\pi + 4} \cong 0,87980 \tag{22}$$

Propulsion force will be equal with $C_x^P \cdot S$, where S is plate aria.

2.2 The problem of the curve plate in unlimited flow

Let us consider that the unlimited fluid encounters the symmetrical, curvilinear, upstream convex obstacle (BOB'). The free lines $(BC),(B'C')$ are detached in the points B,B' and will be infinitely downstream parallel to the Ox axis. Our purpose is to find the shape of the obstacle with maximal drag. The length of the curve $(B'OB)$ is given and is equal to $2L$. These profiles of maximal drag are called "deflectors" or "impermeable parachutes" , and they still correspond to the "Helmholtz model". They are very important in relation with applications to the thrust reversal devices or the direction control of the reactive vehicles. We notice also other applications to the slowing by fluid jets or to the jet flaps systems from the airplanes wings. Within the hypothesis of Section 3 and V^0, L being given we ask the condition of maximum P in (26) and we want to determine the distribution of the velocity on the profile (OB), i.e. $V=V(\xi)$ or $t=t(\xi), \xi\in(-1,1)$. The resultant P [3] is,

$$P=\frac{i\rho {V^0}^2 L}{2}\frac{\oint_{K_\zeta} e^{\omega(\zeta)}d\zeta}{\int_{-1}^1 e^{t(s)}ds}=\frac{\rho {V^0}^2 L}{2\pi}\frac{\left(\int_{-1}^1\frac{t(s)}{\sqrt{1+s}}ds\right)^2}{\int_{-1}^1 e^{t(s)}ds} \tag{23}$$

We write: $P=\frac{\rho {V^0}^2 L}{2}J[t]$ where the nonlinear functional

$$J[t]=\frac{\left(\int_{-1}^1\frac{t(s)}{\sqrt{1+s}}ds\right)^2}{\int_{-1}^1 e^{t(s)}ds} \tag{24}$$

must be maximized. To assure the convergence of the integrals and using (17) and $V(\xi=-1)=0$ we put $V(\xi)=\frac{(1+\xi)^\alpha}{2}g(\xi)$ with $0<\alpha<1$ and $g(-1)\neq 0$. Without losing the generality, we choose $\alpha=\frac{1}{2}$. From $t(\xi)=\ln\frac{V^0}{V(\xi)}$ we have

$$t(\xi)=G(\xi)+\ln\sqrt{\frac{2}{1+\xi}}, \xi\in(-1,1), \tag{25}$$

where the term $G(\xi)$ is generated by $g(\xi)$. Introducing (25) in (24) we get

$$J[G]=\frac{\left[\int_{-1}^1\frac{G(s)}{\sqrt{1+s}}ds+2\sqrt{2}\right]^2}{\sqrt{2}\int_{-1}^1\frac{e^{G(s)}}{\sqrt{1+s}}ds} \tag{26}$$

In order to find $G(\xi)$, the functional $J[G]$ is maximized to a functional $H[G](J[G]\leq H[G])$ whose maximum point may be easily computed and where the two functionals have the same value. The Jensen's inequality [7] is: if $f(x)\geq 0, g(x)$ are integrable functions on $[a,b]$, then

$$\int_a^b f(x)e^{g(x)}dx\geq\left(\int_a^b f(x)dx\right)e^{\frac{\int_a^b f(x)g(x)dx}{\int_a^b f(x)dx}} \tag{27}$$

where the equality case holds if and only if $g(x)$ is a constant function. Applying the Jensen's inequality for (26) we have

$$J[G]\leq\frac{2(U+1)^2}{e^U}\equiv H[U(G)], U(G)\equiv\frac{\sqrt{2}}{4}\int_{-1}^1\frac{G(s)}{\sqrt{1+s}}ds. \tag{28}$$

In (28), the equality holds if and only if $G(\xi) \equiv G_0 = const.$ The functional $H[U]$ has a maximum point in $U_0 \equiv 1$, obtained by differentiation and $H_{max} = H[U_0 = 1] = \frac{8}{e}$. For $G(\xi) \equiv G_0 = 1$ in (28) we have equality and then $J_{max} = J[G_0 = 1] = \frac{8}{e}$. Using (25) we obtaine:

$$t(\xi) = 1 + \ln\sqrt{\frac{2}{1+\xi}}, V(\xi) = \frac{V^0}{e}\sqrt{\frac{1+\xi}{2}}, \xi \in (-1, 1) \tag{29}$$

The result will be $P_{max} = \frac{\rho {V^0}^2 L}{2\pi} J_{max} = \frac{4\rho {V^0}^2 L}{\pi e}$. The maximal drag coefficient will be $C_x^{II} = \frac{2P}{\rho {v^0}^2 L}$, i.e.

$$C_x^{II} = \frac{8}{\pi e} = 0,936797. \tag{30}$$

This result is in agreement with that obtained in [6] by Maklakov using the Levi-Civita method.

2.3 The deduction of the plate sail for maxim lift in a wind parallel with profil chord

We consider a simmetrical curve plate (AB) axa x'x on profile chord and O in the midle of the chord and the wind is paralel with the chord. Let be $L(AB)$ length of (AB) and l length of chord knowns again A_0A, BB_0 free lines with A_0AMBB_0 stream line $\psi = 0$. We denote by $k = \frac{L-l}{l}$ and we will must to determine optimal geometrical shape for maximum lift P(rectangular on chord)[6]. We consider T1,T2 theorems with integral equations (14),(15) and we will determine potential function $f = f(\zeta)$ and $\bar{\omega} = \bar{\omega}(\xi)$ in the upper half plane, $\eta \geq 0$; the plate (AB) being lateral acting of wind with the speed $V^0\vec{i}$. Let be $f(\zeta)$ complex potential and $\bar{\omega} = \frac{df}{dz} = \frac{df}{d\zeta}\frac{d\zeta}{dz}$,

$$f(\zeta) = AV^0\zeta; dz = \varphi\xi e^{i\theta}d\xi, \psi = 0, \eta = 0 \tag{31}$$

$$z(\xi) = \int_{-1}^{\xi} \varphi_\xi \frac{e^{i\theta}}{V(\xi)}d\xi, dS = \varphi_\xi \frac{d\xi}{V(\xi)} \tag{32}$$

$$L = AV^0 \int_{-1}^{1} \frac{d\xi}{V(\xi)} = A\int_{1}^{1} e^{t(S)}dS. \tag{33}$$

The resultant of presures is

$$X + iY = i\rho {V^0}^2 A \oint e^{\omega(\zeta)}d\zeta \tag{34}$$

and because the simetry, $X = 0, Y = \rho {V^0}^2 A \int_{-1}^{1} t(S)dS$.
The lift will be:

$$Y = \rho {V^0}^2 LJ(t); J(t) = \frac{2\int_{-1}^{1} t(S)dS}{\int_{-1}^{1} e^{t(S)}dS} \tag{35}$$

Applying the Jensen's inequality at denominator of $J \leq I$, we search the velocity distribution on (AB) so that the functional J(t) to be maximum; if we note $H = \int_{-1}^{1} t(S)dS$ we will obtain $J \leq I = He^{-\frac{H}{2}}$ in the case equal the functional is $I_{\max}$.

For the maximum, the derived $I'(H) = 0$, with $H \succ 0, I' = e^{-\frac{H}{2}}\left(1 - \frac{H}{2}\right)$. For $H = 2, I_{\max} = \frac{2}{e}$ and $t(\xi) = 1$.With (14),(12) we obtain

$$A = \frac{L}{2e}, V = \frac{V^0}{e}, \theta(\xi) = -\frac{\sqrt{\xi+1}}{\pi}\int_{-1}^{\xi}\frac{dS}{\sqrt{S+1}(S-1)} =$$

$$= \frac{1}{\pi}\ln\frac{\sqrt{2}+\sqrt{1+\xi}}{\sqrt{2}-\sqrt{1+\xi}}, \xi \in (-1, 1)$$

and with $\frac{l}{L} = \int_{-1}^{1}\cos\theta(\xi)d\xi = \frac{2e}{e^2-1}, k = sh(e-1)$ [6].
From Y with I_{max} we obtain the lift coefficient $C_y = \frac{Y}{\rho V^{0^2} L} \leq C_{y\max}, C_{y\max} = \frac{2}{e}(1+k)$. The optimal lift for plate will be

$$P_{\max} = C_{y\max} \cdot S, k = 0,175, C_{y\max} \approx 0,876.$$

Wu and Whitney have study this problem [6] with application at the flight of "para-slope".Also, Maklakov [6]has find this solution .

3 The problem of sail with flaps assimilated to a point-vortex

We consider the sail with an airfoil kind shape, having one third of the extrados toward the one flap attack board. Between the sail and the flap there is a breach through which a part of the air flow is passing. Since the breach is narrow the air flow speed through it is greater than the speed of the incidental flow on the sail and according to Bernoulli's relationship, the pressure is dropping on the extrados and a certain raise of the sail lift appear.
We are considering the circulation movement around the sail and around the flap. This flap can be simulated by a vortex placed in the vicinity of the attack board toward the extrados. If the Jukovski transformation is applied, the profile becomes a circle in the (Z) plane and we have a translation potential $f_1(Z)$; hence the vortex potential is $f_2(Z) = \frac{I_0}{2\pi i}\ln(Z - Z^*)$,in the presence of the circle. We obtain, with the help of Caius Iacob [1], [3] extended circle theorem, for sources and vortexes, the full potential of the incompressible fluid movement in the presence of the circle. By applying the Jukovski theory stating that in the stagnation points on the circle we must have zero speed, we obtain the flow's value on the profile and the hydrodynamic resultant. [1], [2], [3]. We consider the sail profile in a form of a thin ellipse.
We use the conformal representation principle to determine the complex potential of the fluid's movement which, at great distances, has the speed $V_0 e^{-i\alpha}$, in the presence of an obstacle with the exterior that we know to represent conformal on a circle exterior. In the obstacle vicinity in the second quarter there is a vortex having the intensity I, placed in a point z_1; the obstacle can have an angular point at the run board. The conformal transformation $Z = Z(z)$, according to the known theory, fulfils the conditions $Z(\infty) = \infty, Z'(\infty) = \lambda$ [1].
We consider the translation movement having the potential $f_1(Z) = V_0 e^{-i\alpha} Z$ and the vortex with $f_2(Z) = \frac{I}{2\pi i}\ln(Z - Z_1)$, Z_1 exterior to the circle C. Given $f_0(Z) = f_1(Z) + f_2(Z) + f_3(Z)$, where $f_3(Z) = \frac{\Gamma}{2\pi i}\ln Z$, Γ being the flow on the circle $|z| = R$.

Knowing I, Γ can be determine from the study of the speed on the circle in the points corresponding to the stagnation points z^* on the profile [1], [3]. Hence, determining the complex potential $F = F(Z)$ and the speed $W = \frac{dF}{dZ}\frac{dZ}{dz}, W_p = W_c \left(\frac{dz}{dZ}\right)^{-1}$, where $W_p(z^*) = 0$ the flow Γ can be obtain by placing the condition $\frac{dz}{dZ}\mid_C = 0$ [3]. If we assume that the circle C has the center in the origin ,$f_1(Z) = V_0 e^{-i\alpha} Z$ and $F_0(Z) = \frac{\Gamma}{2\pi i}\ln Z + \frac{I}{2\pi i}\ln(Z - Z_1)$, then the complex potential $F(Z)$ which has the same singularities as F_0 and f_1, with $\Im F(z)\mid_C = 0$, is [3]

$$F(Z) = f_1(Z) + \overline{f_1\left(\frac{R^2}{\bar{Z}}\right)} + F_0(Z) + \overline{F_0\left(\frac{R^2}{\bar{Z}}\right)} - \frac{\Gamma}{2\pi i}\ln\frac{Z}{R} + C, C \in R \tag{36}$$

$$F(Z) = V_0 e^{-i\alpha} Z + \frac{V_0 e^{i\alpha}}{Z} + \frac{\Gamma + I}{2\pi i}\ln Z + \frac{I}{2\pi i}\ln\frac{Z - Z_1}{Z - Z_1'} + C_1 \tag{37}$$

$$\frac{dF}{dZ} = V_0\left(e^{-i\alpha} - \frac{e^{i\alpha}}{Z^2}\right) + \frac{\Gamma + I}{2\pi i}\frac{1}{Z} + \frac{I}{2\pi i}\left(\frac{1}{Z - Z_1} - \frac{1}{Z - Z_1'}\right) \tag{38}$$

The stagnation points can be obtained from a forth degree equation. For the ellipse $Z = \frac{z + \sqrt{z^2 - c^2}}{a+b}$ having the plate $b = 0$.

4 A numerical method to determine the propulsion force obtained from wind energy by a rigid sail with aerodynamic profile

We suppose that the lift and drag coefficients for aerodynamic profile of sail are known functions of incidence angle,$C_L(i), C_D(i)$, respective.

We consider that the motion direction of ship is reference axis and the position of sail and wind direction in relation with that are known by the angles β andγ_∞. In sail's neighborhood wind velocity is smaller than at the great distance, $U \prec V_\infty$. We apply the current tubes method for to calculate U on the all sail's height [9]. We divide the sail in elements with the length Δh . We suppose rectangular form for the current tube with sizes Δh and $C_k \sin(\gamma_\infty - \beta)$, where C_k is chord's length of sail aerodynamic profile for k element. If the ship's velocity is $\vec{V_n}$ then the relative velocity of wind is the result of $\vec{U}$ and $-\vec{V_n}, \vec{U_r} = \vec{U} - \vec{V_n}$. The incidence angle and U_r are:

$$\prec i = arcctg\frac{U\cos\gamma_\infty + V_n}{U\sin\gamma_\infty} - \beta \tag{39}$$

$$U_r = \sqrt{U^2 + V_n^2 + 2UV_n\cos\gamma_\infty} \tag{40}$$

The aerodynamic forces what operate in a current tube and on the sail's element are: $\vec{L}$ (lift), $\vec{D}$ (drag), $\vec{F_t}$ (drive force), $\vec{F_\infty}$ (element's force on air in tube). The expresion of those forces are:

$$L = \frac{1}{2}\rho C_k \Delta h C_L U_r^2; D = \frac{1}{2} C_k \Delta h C_D U_r^2;$$

$$F_t = L\cos x + D\sin x; F_\infty = L\cos(\gamma_\infty - x) - D\sin)\gamma_\infty - x)$$

where $x = -arctg\frac{U\cos\gamma_\infty + V_n}{U\sin\gamma_\infty}$. The Force of air current in tube on sail's element is: $F_a = \rho C_k \sin(\gamma_\infty - \beta)\Delta h U(V_\infty - u)$. From equality of F_∞ and F_a the relation for the velocity in current tube result:

$$2\sin(\gamma_\infty - \beta)\frac{U}{V_\infty}\left(1 - \frac{U}{V_\infty}\right) = \left[\left(\frac{U}{V_\infty}\right)^2 + \left(\frac{V_n}{V_\infty}\right)^2 + 2\frac{UV_n}{V_\infty^2}\cos\gamma_\infty\right]\cdot$$
$$\cdot\left[C_L\cos(\gamma_\infty - x) - C_D\sin(\gamma_\infty - x)\right] \quad (41)$$

The equation (41) is nonlinear for $\frac{U}{V_\infty}$. We apply an iterative method for solving it. We consider a new variable, $a = 1 - \frac{U}{V_\infty}$ in (41)and from the obtained relation separate "a" of the rest: $a = f(a)$. The relation of iteration "n" and "n+1" is: $A^{(n+1)} = f\left(a^{(n)}\right), a^{(1)} = 0$. If $\left|a^{(N+1)} - a^{(N)}\right| \leq \varepsilon$ (given) then $a \approx a^{(N+1)}$. We calculate now U, x, i, L, D and F_t for k element. The drive force on the all element is: $F = \sum_{k=1}^{N1}(F_t)_k$

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